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THE RIGIDITY OF THE EARTH

A. A. MICHELSON AND HENRY G. GALE

In 1914, in the March number of the *Astrophysical Journal* (**39**, 105, 1914), an account was published of a preliminary experiment to determine the rigidity of the earth. At that time it was announced that the experiment would be repeated, using an interference method. The new arrangements were completed and a new series of observations begun on November 20, 1916, and continued until November 20, 1917. The reduction of the observations was interrupted by the war in the summer of 1917 and could not be resumed until April 1919.

The same pits and pipes on the grounds of the Yerkes Observatory at Williams Bay, Wisconsin, were used as in the preliminary experiment. In that experiment pipes 502 feet long and 6 inches in diameter were placed 6 feet underground. One pipe was laid accurately N–S and the other E–W. The pipes ended in pits 10 feet deep and 8 feet square, walled with concrete. The pipes were carefully leveled, and half filled with water, so that an air space extended from end to end of each pipe, above the water. The pipes ended in air-tight gauges provided with windows through which the changes in water-level could be determined by measuring with microscopes the distance between pointers just below the surface of the water and their totally reflected images.

In the present experiment interferometers replaced the microscopes and pointers. The arrangement of the interferometers, one at each end of each pipe, is shown in Figure 1. The compensating glass serves to seal the pipe. The lower mirror is movable vertically and has also the usual adjustments for regulating the width and orientation of the fringes. The film of water over this mirror is kept thin, usually about 0.5 mm, as the viscosity of the water helps to dampen ripples and minor disturbances. The changing thickness of the water film, due to the tides, caused the shift of fringes. The arrangement for recording the fringes was as follows: Horizontal fringes were projected by the lens *L* on a narrow vertical slit about 0.2 mm in width. Clockwork drew a moving-picture film behind this slit at the rate of about 2 cm per hour. In order to prevent the condensation of moisture on the optical parts the end of the pipe, interferometer, and camera were all inclosed in a galvanized iron box, in which large trays of calcium chloride were kept. An incandescent light was continually burning near the interferometer to keep the temperature slightly raised.

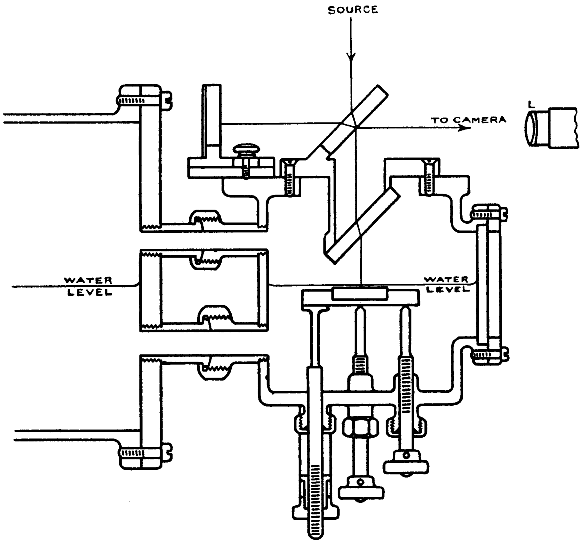


FIG. 1

Other interference arrangements are obvious which would give a displacement of a greater number of fringes, or permit the use of a shorter pipe; e.g., the fringes formed between the water surface and the lower mirror might be used, or the lower mirror might be dispensed with and use made of the fringes formed by the light reflected from the water surface and the vertical mirror. But the arrangement actually used was the most satisfactory, since the long pipes, 502 feet, were already installed.

The sources of light were commercial alternating-current Cooper-Hewitt mercury lamps. They proved very reliable and satisfactory. By using as filters thin films of a saturated solution of esculin in water, all wave-lengths from the arc shorter than λ 4358 were absorbed, and the positive film used was not sensitive to the longer wave-lengths. The exposed portions of the films were removed and developed each week. The light was abundantly strong for satisfactory negatives, and it was possible to use 1.5 mm diaphragms on the projecting lenses. This gave sufficient sharpness to the fringes, even when there was a considerable change in their focus. It was necessary to readjust and refocus the fringes in only one pit during the entire year, although the width of the fringes was altered once or twice in two other pits. One of the mirrors required resilvering. One of the pits ran throughout the year without readjustment of the fringes or camera. The pits and cameras were in charge of Mr. George Monk and Mr. Frank Sullivan, of the Yerkes Observatory staff.

A relay which moved a shutter in front of the projecting lens was placed in each pit. The four relays were connected in series with a clock in the observatory, so that the time could be conveniently and accurately controlled. Once an hour the clock made contact, and a storage-battery circuit was closed through the relays and the light was cut off by the shutters for about 20 seconds. Very accurate time-marks were secured in this way. The control clock was kept six minutes faster than Central Standard Time in order to simplify the computations and bring the observations into agreement with them. (The longitude of Yerkes Observatory from Greenwich is $5^h\ 54^m\ 13^s$.)

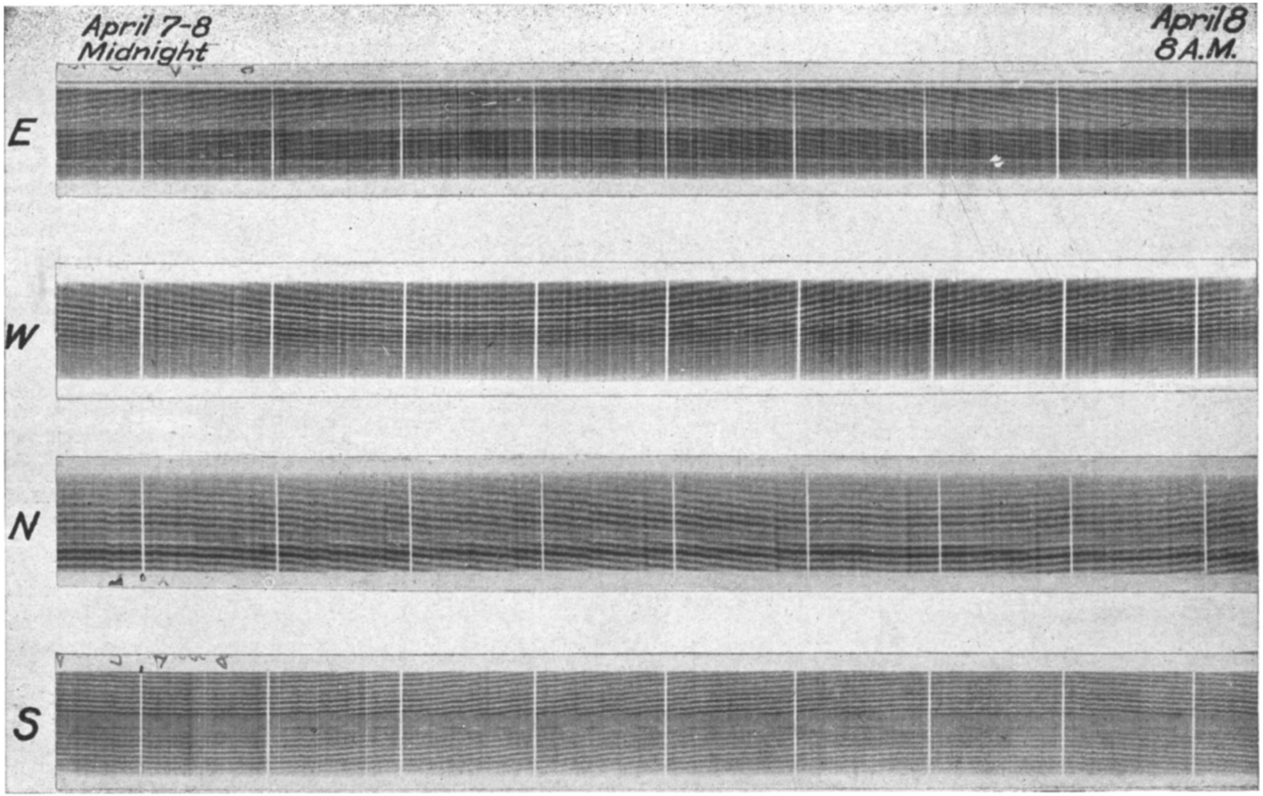
The films were measured by sliding them on a lathe-bed beneath a low-powered microscope. The fringes, estimated to tenths, were counted as they moved up and down, and the numbers recorded for each hour. The difference in the motion at the two ends of each pipe gave the numbers for plotting the observed tides.

The calculated tides were drawn from the computed shift in fringes, the calculations being made for two-hour intervals. The calculations were made under the direction of Professor F. R. Moulton by Mr. Albert Barnett and Mr. Horace Olsen. The formulae are given in the accompanying article by Professor Moulton, "Theory of Tides in Pipes on a Rigid Earth." The value of μ for the water used was found to be 1.3408 for λ 4358, and this is probably correct to within considerably less than 0.1 per cent for the range of temperatures used.

Calculated and observed curves for the period from March 24 to April 21, 1917, are reproduced in Figures 2 and 3. The dotted curve represents the observed and the full curve (displaced vertically to avoid overlapping) 0.7 of the calculated values of the tides. The ordinates are numbers of fringes, $N=\frac{2(\mu-1)d}{\lambda}$, and one fringe corresponds to 1/1564 mm.

The observed and calculated curves were plotted on long rolls of co-ordinate paper to the following scale: abscissae, 1 cm = 1 hour; ordinates, 1 cm = 2 fringes. In order to have the amplitudes approximately equal, 0.7 of the calculated values were plotted instead of the full amplitudes. Beginning with 10:00 A.M. November 20, 1916, the curves, both observed and calculated, were divided into periods of $12^{\text{h}}.42$ for the semi-diurnal and $25^{\text{h}}.82$ for the diurnal lunar tides. The principal solar tide, period twelve hours, was started at noon of the same day. In order to avoid a cumulative error in the case of the semi-diurnal lunar tide the period $12^{\text{h}}.4206013$ was put on a computing machine and added repeatedly to the initial time to get the exact beginning of each new period throughout the year. This process was repeated, using the period $25^{\text{h}}.8193409$ for the diurnal lunar tide.

The observations were reduced in groups of about a lunar month each, by dividing each period into ten equal parts (twelve



PHOTOGRAPHIC RECORDS TAKEN SIMULTANEOUSLY in the case of the solar tide), and taking the mean of the first, second, third, etc., ordinates. The resulting values were plotted and any error in computation was usually indicated by the failure of a point to fall on a smooth curve. It is important to treat the observed and calculated tides both in the same way, as any distortion in the resulting sine curves due to lack of complete elimination of other periods affects the two alike. This is, of course, most noticeable in the case of the diurnal tide on account of its smaller amplitude and the smaller number of periods. Mr. Fred Pearson gave valuable assistance in measuring the films, in plotting the curves, and in deducing the various tides from the curves.

Very little trouble was caused by sudden erratic changes in the fringes. Occasionally, however, earthquakes would cause the fringes to disappear for from ten minutes to half an hour. Once the effects of an earthquake were evident for about six hours. During three hours of this time the fringes were completely obliterated.

The most serious disturbance was a gradual change in the slope of the observed curves. This would often be fairly uniform and gradual for a month or two. At some times the curves would rise and at others fall. Sometimes the N–S and E–W slopes had the same sign, and sometimes opposite signs. We have been able to discover nothing systematic about this drifting. It may have been caused by unequal settling at the ends of the pipes, by temperature changes in the pits, or by tilting in the earth's strata. There were always large shifts of the fringes when the lights came on after having been interrupted by the power company for a half-hour or so. The change of slope was eliminated in reducing each monthly tide, as given in Tables I–VI, and the tide for the whole year, opposite *Y* in the tables, by distributing the change of level uniformly throughout the period. This change of slope is quite conspicuous in Figure 3, where the observed E–W tide showed a fairly uniform and distinct downward trend throughout nearly the whole month. The change in slope of the N–S curves for the same period is comparatively small, as shown in Figure 2.

Plate III is from four photographs taken simultaneously in the four pits. The reproductions are positives on the same scale

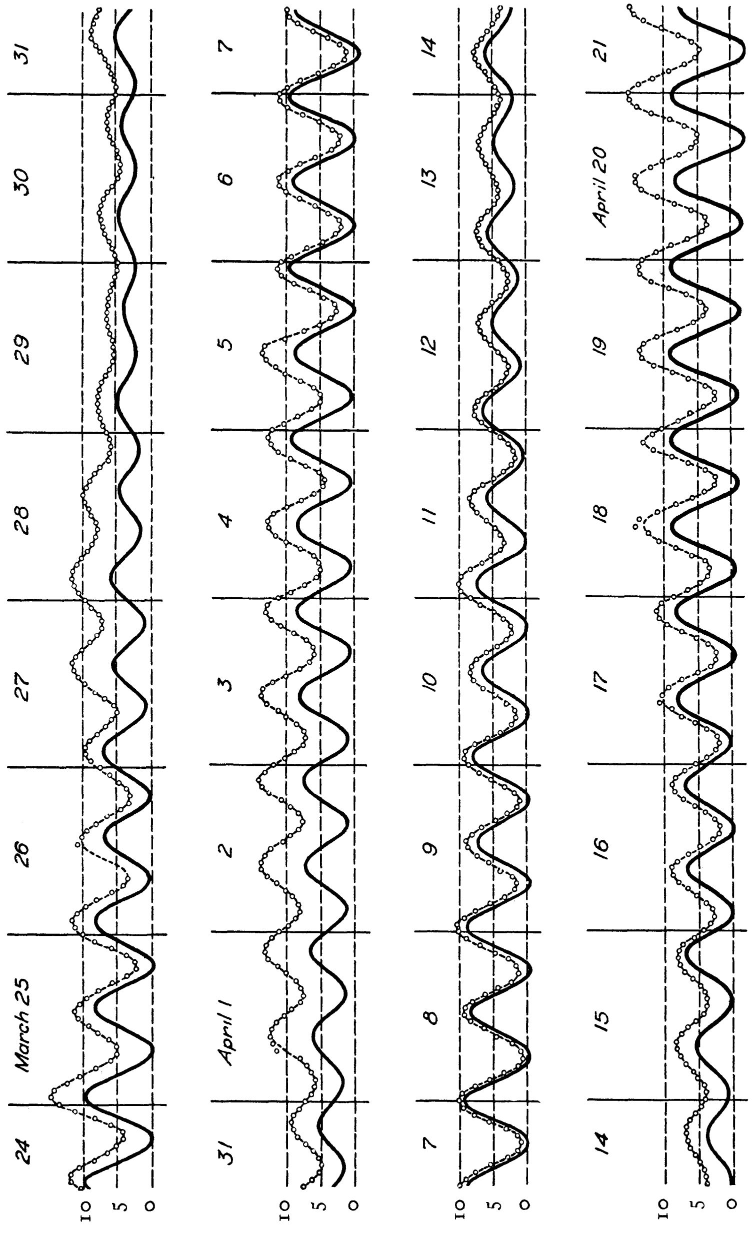


FIG. 2.—N-S tides, March 24 to April 21, 1917. Dotted curves, observed values. Full curves, 0.7 of calculated values

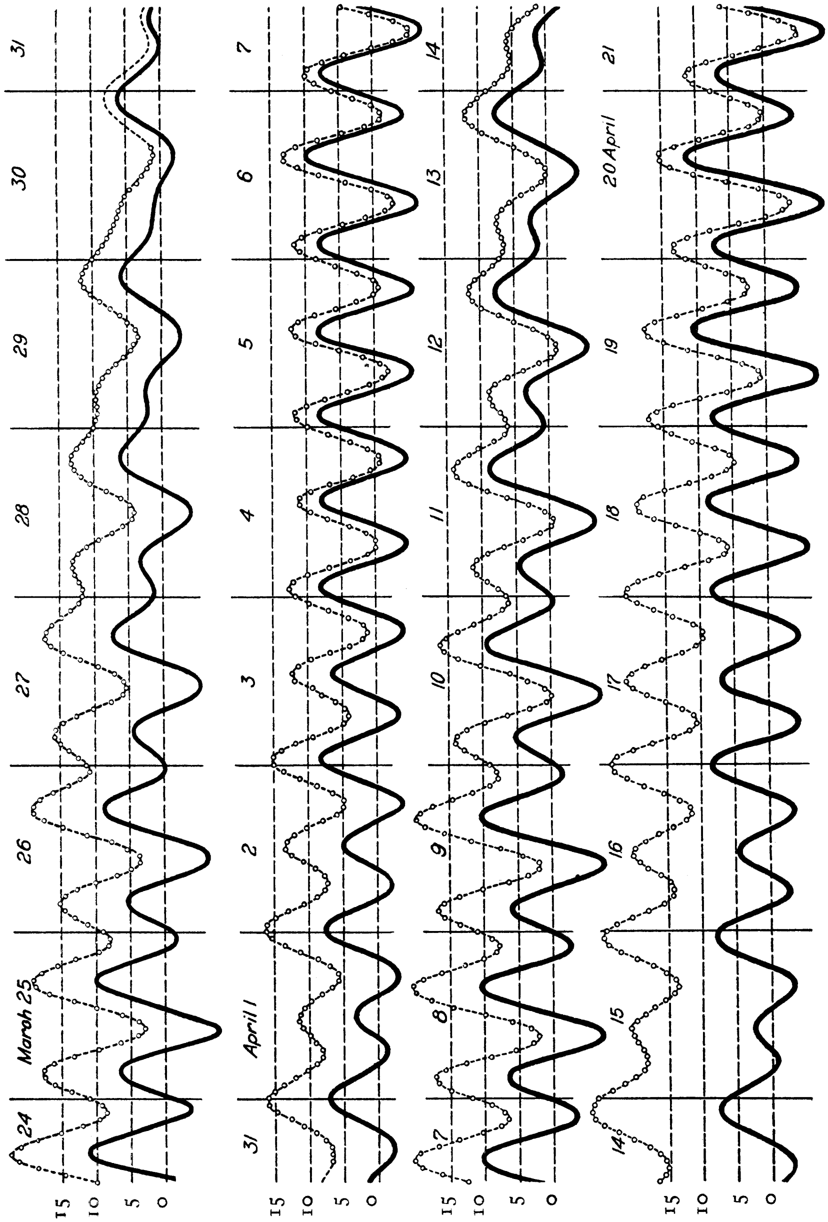


FIG. 3.—E–W tides, March 24 to April 21, 1917. Dotted curves, observed values. Full curves, 0·7 of calculated values

as the originals, and represent fairly well the average quality of the films.

A graphical solution is excellent for detecting erroneous points and serves well to give the ratio of the observed to the calculated amplitude, but for determining the phase-difference of the two curves it is not so satisfactory. A least-squares method was therefore used to secure the ratio of amplitudes, *R*, and the displacement in phase $\Delta\phi$ of the observed with respect to the calculated tide. The following example illustrates the method of reduction. It is for the semi-diurnal tide, first month, E–W. For convenience the solar periods were divided into twelve parts instead of ten, but in other respects the method of reduction is the same.

In Tables I–VI the numbers in the columns under *M* denote the different approximate lunar months; under *N* is given the number of periods used. For the solar tides missing portions of the observed curves were sketched in, following the computed tides, thus giving a total of 730 periods for the year, but for the semi-

FIRST MONTH, SEMI-DIURNAL, E–W OBSERVED

θ	y	Sin θ	y Sin θ	Cos θ	y Cos θ
0°........	17.58	0.0000	0.0000	1.0000	17.5800
36	18.74	.5878	11.0153	0.8090	15.1606
72	15.73	.9511	14.9608	.3090	4.8605
108	10.46	.9511	9.9485	− .3090	−3.2321
144	4.83	.5878	2.8390	− .8090	−3.9074
180	0.60	.0000	0.0000	−1.0000	−0.6000
216	−0.20	−.5878	0.1234	−0.8090	0.1698
252	2.58	−.9511	−2.4538	− .3090	−0.7972
288	7.76	−.9511	−7.3805	.3090	2.3978
324	13.48	−.5878	−7.9235	.8090	10.9053
Total.....	....	....	+21.1292	....	+42.5373

$$\alpha=\frac{21.1292}{5}=+4.2258 \quad \beta=\frac{42.5373}{5}=+8.5075$$

$$\tan\phi_c=\frac{\beta}{\alpha}=2.0132$$

$$\phi_c=63°35'$$

$$c=\sqrt{\alpha^2+\beta^2}$$

$$c=\sqrt{90.2350}$$

$$c=9.499$$

$$y=9.499 \sin(\theta+63°35')$$

FIRST MONTH, SEMI-DIURNAL, E-W CALCULATED

θ	y	Sin θ	y Sin θ	Cos θ	y Cos θ
0°........	17.38	0.0000	0.0000	1.0000	17.3800
36	18.89	.5878	11.1035	0.8090	15.2820
72	16.73	.9511	15.9119	.3090	5.1695
108	11.58	.9511	11.0137	− .3090	−3.5782
144	6.13	.5878	3.6032	− .8090	−4.9591
180	0.90	.0000	.0000	−1.0000	−0.9000
216	0.00	−.5878	.0000	−0.8090	.0000
252	2.23	−.9511	−2.1209	− .3090	− .6890
288	7.16	−.9511	−6.8098	.3090	2.2124
324	12.91	−.5878	−7.5885	.8090	10.4441
Total.....	....	....	+25.1131	....	+40.3617

$$\alpha=\frac{25.1131}{5}=+5.0226 \quad \beta=\frac{40.3617}{5}=+8.0723$$

$$\cdot \tan \phi_d=\frac{\beta}{\alpha}=+1.6072$$

$$\phi_d=58°7'$$

$$d=\sqrt{\alpha^2+\beta^2}$$

$$d=\sqrt{90.3885}$$

$$d=9.507$$

$$y=9.507 \sin (\theta+58°7')$$

$$R=\frac{c}{d}\frac{0.7\times 9.499}{9.507}=0.699 \qquad \Delta\phi=\phi_c-\phi_d=+5°28'$$

diurnal and diurnal lunar tides only such portions of the observed curves were used as gave complete periods. Under a and c are given the amplitudes and under ϕ_a and ϕ_c the phase constants for the observed curves and under b and d and ϕ_b and ϕ_d the corresponding quantities for the calculated curves. In taking the means the value for each month is weighted in proportion to the number of periods in the month. Below the mean R and $\Delta\phi$ in each case is given the average difference from the mean. At the bottom of each table opposite Y are given the results obtained by computing the tides for the entire year as a single period instead of for a month at a time. The results agree closely with the means for the thirteen months, and are shown graphically in Figures 4 to 8.

Violent storms broke down the electric wires during February and March on several occasions, interrupting the electric current for a few hours. The pits cooled, and there were resulting large shifts of the fringes after the current was re-established. It seems highly probable that the large difference of phase which is shown for the fourth month E–W diurnal tide is due to such disturbances.

TABLE I

N–S, Semi-Diurnal, Lunar

M	*N*	*a*	*b*	ϕ_a	ϕ_b	*R*	$\Delta\phi$
1......	60	6.217	6.373	149°14′	147°48′	0.683	1°26′
2......	54	5.703	6.489	147 53	144 4	.615	3 49
3......	48	7.261	6.780	146 15	145 56	.750	0 19
4......	45	6.750	6.739	148 44	146 23	.701	2 7
5......	55	6.041	6.269	148 37	148 54	.675	−0 17
6.....	54	6.426	6.551	148 35	147 57	.687	0 38
7......	54	6.332	6.344	148 40	145 56	.699	2 44
8......	51	6.052	6.344	147 47	146 58	.668	0 49
9......	55	6.036	6.319	146 36	147 16	.669	−0 40
10......	54	5.821	6.210	149 27	147 32	.656	1 55
11......	54	5.875	6.140	148 14	148 16	.670	−0 2
12......	55	6.027	6.284	149 18	147 46	.671	1 32
13......	45	6.330	6.515	152 39	150 36	0.680	2 3
Av....	....	6.203	....	....	....	0.678 ±.019	1°15′ ±1 2
Y	684	6.173	6.401	148°24′	147°32′	0.675	0 52

TABLE II

E–W, Semi-Diurnal, Lunar

M	*N*	*c*	*d*	ϕ_c	ϕ_d	*R*	$\Delta\phi$
1......	60	9.499	9.507	63°35′	58° 7′	0.699	5°28′
2......	54	8.908	9.096	64 40	58 5	.686	6 35
3......	48	9.899	10.075	61 28	55 47	.688	5 41
4......	43	10.220	10.369	69 46	65 2	.690	4 44
5......	56	9.131	9.380	63 43	57 0	.681	6 43
6......	53	9.660	9.667	65 4	57 54	.700	7 10
7......	54	9.496	9.632	64 26	57 6	.690	7 20
8......	51	9.445	9.331	60 48	55 4	.709	5 44
9......	55	9.206	9.316	63 24	57 30	.692	5 54
10......	54	8.773	9.095	64 52	56 32	.675	8 20
11......	54	8.982	9.108	61 31	57 18	.690	4 13
12......	55	9.138	9.337	63 12	58 12	.685	5 0
13......	45	9.809	9.747	67 20	61 44	0.704	5 36
Av....	....	9.373	....	....	....	0.691 ±.007	6° 4′ ± 55
Y	682	9.380	9.485	63°56′	58°10′	0.692	5 46

Moreover, this month, on account of the interruptions, contained but 20 periods instead of the usual 26. The mean is therefore given for twelve lunar months, omitting the fourth, on the line Av_{12} as

TABLE III

N–S SOLAR

M	N	a	b	ϕ_a	ϕ_b	R	$\Delta\phi$
1......	56	2.554	2.628	104° 8′	99°39′	0.680	4°29′
2......	62	2.392	2.294	100 13	85 45	.730	14 28
3......	48	4.883	4.272	93 34	82 59	.800	10 35
4......	70	2.778	3.132	88 41	80 35	.621	8 6
5......	58	3.840	3.744	100 26	97 14	.718	3 12
6......	54	3.595	3.485	102 43	99 59	.722	2 43
7......	56	2.818	2.570	109 29	103 27	.768	6 2
8......	54	2.513	2.312	109 1	99 49	.761	9 12
9......	56	2.771	2.506	87 23	84 18	.774	3 5
10......	56	2.944	3.108	89 35	86 54	.663	2 41
11......	56	3.562	3.456	99 10	95 51	.721	3 19
12......	56	3.154	3.291	108 6	103 40	.671	4 26
13......	48	3.775	3.889	98 55	99 38	0.679	−0 43
Av....	....	3.161	....	....	....	0.714 ±.041	5°38′ ±3 16
Y....	730	3.140	3.063	99°25′	93°20′	0.718	6 5

TABLE IV

E–W SOLAR

M	N	c	d	ϕ_c	ϕ_d	R	$\Delta\phi$
1......	56	3.641	3.838	13°54′	9°40′	0.664	4°14′
2......	62	3.218	3.400	3 31	355 40	.663	7 51
3......	48	6.172	6.488	358 28	352 59	.666	5 29
4......	70	4.444	4.665	354 5	350 14	.667	3 51
5......	58	5.534	5.480	11 19	6 33	.707	4 46
6......	54	5.104	5.173	16 52	10 59	.691	5 53
7......	56	3.687	3.731	21 32	13 53	.692	7 39
8......	54	3.487	3.407	15 3	8 22	.716	6 41
9......	56	3.450	3.689	359 50	353 28	.655	6 22
10......	56	4.371	4.595	3 14	356 36	.666	6 38
11......	56	5.045	4.993	7 57	4 32	.707	3 25
12......	56	4.689	4.858	16 28	14 16	.676	2 12
13......	48	5.621	5.626	10 0	6 11	0.699	3 49
Av....	....	4.459	....	....	....	0.681 ±.018	5°19′ ±1 28
Y....	730	4.399	4.522	8° 6′	2°23′	0.681	5 43

TABLE V

N-S, Diurnal, Lunar

M	N	a	b	ϕ_a	ϕ_b	R	$\Delta\phi$
1......	26	0.962	0.551	185°35′	193°24′	1.222	− 7°29′
2......	28	0.653	.470	189 59	188 19	0.973	1 40
3......	24	1.306	.550	319 49	201 31	1.662	118 18
4......	20	2.042	.701	305 43	218 48	2.039	86 55
5......	25	0.117	.509	189 00	203 36	0.336	−14 36
6......	26	.347	.465	218 59	197 7	.522	21 52
7......	26	.335	.499	190 27	194 25	.475	− 3 58
8......	25	.279	.484	193 22	193 25	.404	− 0 3
9......	26	.297	.463	213 32	195 58	.451	17 34
10......	26	.538	.473	154 44	193 31	.796	−38 37
11......	26	.277	.455	164 41	194 4	.426	−29 23
12......	26	.282	.365	214 36	188 58	.541	+25 38
13......	22	0.359	0.480	181 1	191 58	0.523	−10 57
Av_{13}...	....	0.574	....	....	....	0.777 ±0.411	11°57′ ±30 49
Av_{12}...	....	0.478	....	....	....	0.694 ±0.313	7 3 ±26 6
Av_{11} ..	....	0.408	....	....	....	.612 ±0.216	− 2 25 ±16 12
Y_{13} ...	326	0.281	0.503	229°23′	197°57′	0.411	31 26
Y_{12} ...	306	0.298	0.492	203 24	196 07	.424	7 17
Y_{11} ...	282	0.411	0.489	188 14	196 56	0.588	− 8 42

TABLE VI

E-W, Diurnal, Lunar

M	N	c	d	ϕ_c	ϕ_d	R	$\Delta\phi$
1......	26	4.107	3.819	282° 9′	284°56′	0.753	−2°47′
2......	28	3.845	3.725	275 48	278 19	.723	−2 31
3......	24	4.164	4.193	288 57	288 58	.695	−0 1
4......	20	3.525	3.603	317 6	293 57	.685	23 9
5......	25	3.925	4.134	287 31	285 43	.665	1 48
6......	26	3.517	3.850	291 19	288 14	.639	3 5
7......	26	3.925	3.900	281 44	285 13	.704	−3 29
8......	25	4.032	3.968	281 54	281 24	.711	0 30
9......	26	3.710	3.518	286 41	283 20	.738	3 21
10......	26	3.480	3.595	282 29	284 21	.678	−1 53
11......	26	3.383	3.442	281 40	283 59	.688	−2 19
12......	26	3.439	3.470	278 57	281 36	.694	−2 39
13......	22	4.526	4.485	275 17	280 35	0.706	−5 18
Av_{13}...	326	3.807	....	....	....	0.699 ±0.022	0°29′ ±4 11
Av_{12} ..	306	3.825	....	....	....	0.700 ±0.024	−1 0 ±2 8
Y_{13} ...	326	3.759	3.799	284°55′	283°22′	0.693	1 33
Y_{12} ...	306	3.807	3.815	283 52	282 43	0.699	1 9

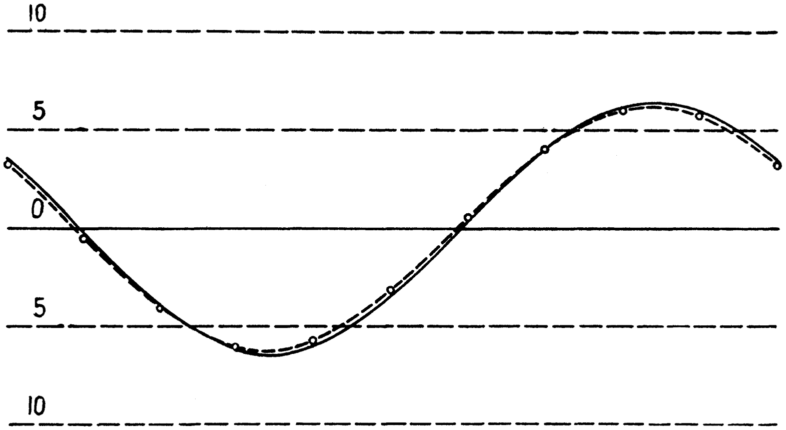


FIG. 4.—N-S semi-diurnal lunar tide for entire year. Dotted curve from observed values

$$y=6.173 \sin (\theta+148°24')$$

Full curve from 0.7 calculated values

$$y=6.401 \sin (\theta+147°32')$$
$$R=0.7 \quad a/b=0.675. \quad \Delta\phi=52'$$

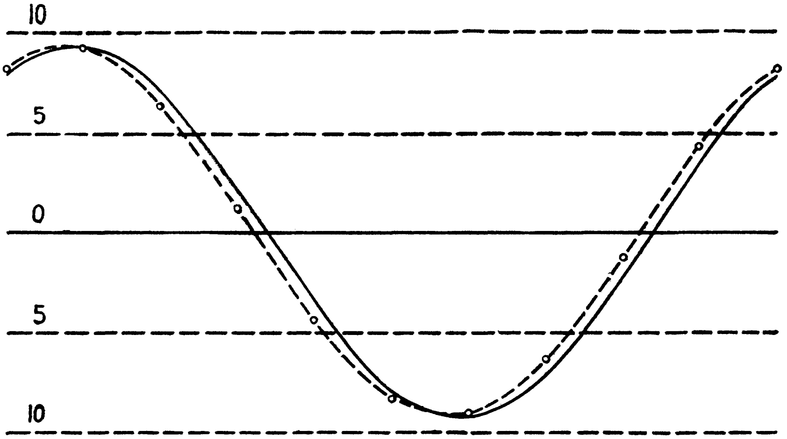


FIG. 5.—E-W semi-diurnal lunar tide for entire year. Dotted curve from observed values

$$y=9.380 \sin (\phi+63°56')$$

Full curve from 0.7 of calculated values

$$y=9.485 \sin (\theta+58°10')$$
$$R=0.7 \quad c/d=0.692. \quad \Delta\phi=5°46'$$

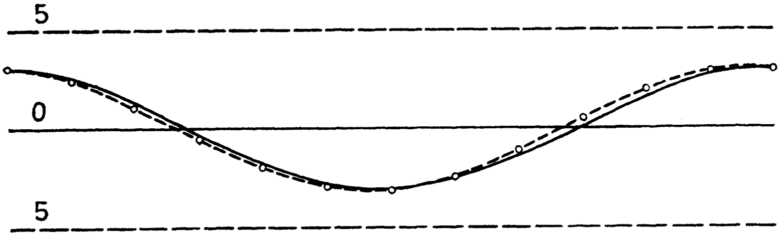


FIG. 6.—N-S solar tide for entire year. Dotted curve from observed values

$$y=3.140 \sin (\theta+99^\circ 25')$$

Full curve from 0.7 of calculated value

$$y=3.063 \sin (\theta+93^\circ 20')$$
$$R=0.7\ a/b=0.718. \quad \Delta\phi=6^\circ 5'$$

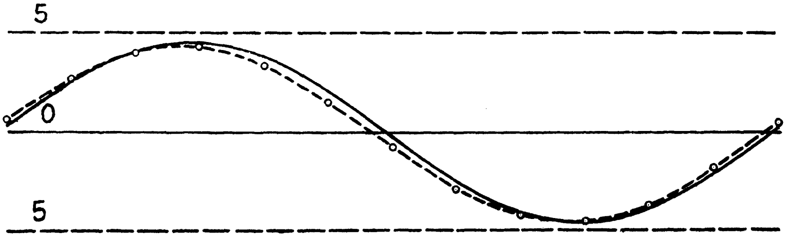


FIG. 7.—E-W solar tide for entire year. Dotted curve from observed values

$$y=4.399 \sin (\theta+8^\circ 6')$$

Full curve from 0.7 of calculated value

$$y=4.522 \sin (\theta+2^\circ 23')$$
$$R=0.7\ c/d=0.681. \quad \Delta\phi=5^\circ 43'$$

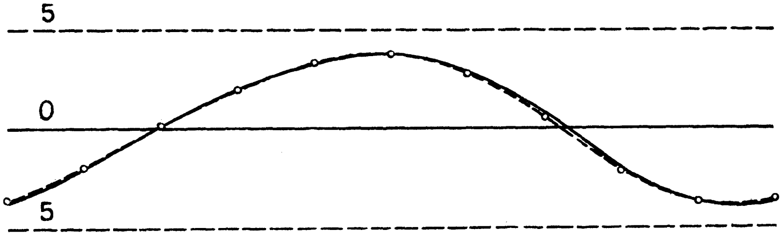


FIG. 8.—E-W diurnal lunar tide for entire year. Dotted curve from observed values

$$y=3.759 \sin (\theta+284^\circ 55')$$

Full curve from 0.7 of calculated values

$$y=3.799 \sin (\theta+282^\circ 53')$$
$$R=0.7\ c/d=0.693. \quad \Delta\phi=1^\circ 33'$$

well as for the thirteen months. A calculation for 306 periods was also made, omitting this month from both the observed and calculated data. This value is given opposite Y_{12}.

In the case of the N–S diurnal tide it will be noted that the amplitude is very small, about 0.5 fringe, as it should be since this tide has the coefficient cos $2l$, where l is the latitude. For Yerkes Observatory cos $2l=0.0848$. This tide is too small to admit of much accuracy in the determination, but the results are included, as they are not without interest. Mean values are added for the whole year, and the means omitting the fourth month, and also omitting both the third and the fourth. The calculations are also added for the year as a whole, 326 periods; omitting the fourth month, 306 periods; and omitting both the third and fourth months, 282 periods. The omission of these months is perhaps justified, since both R and $\Delta\phi$ are decidedly abnormal. The mean of the six values gives $R=0.584$ and $\Delta\phi=7°46'$. Probably the only conclusion which is justified for the N–S diurnal tide is that R is about $0.6\pm .2$ and that the difference of phase is small.

An effort was made to deduce the fortnightly lunar tide of period 13.66 days. Here the E–W tide should be zero, and the residual sine curves were less than 0.05 fringe from both the calculated and observed curves. The N–S tide, however, had an amplitude of about 1.8 fringes and gave $R=0.628$ and $\Delta\phi=-8°24'$, quantities which agree as well as could be expected with the shorter periods. The negative sign of $\Delta\phi$ merely indicates that the uncertainty is considerable.

The results are collected in Table VII. The value given in each case is the average of the mean value for the thirteen lunar months and the value deduced by treating all the observations in a single set, except in the case of the N–S diurnal tide, where the values given are the mean of the six determinations mentioned above, and in the case of the E–W diurnal tide, which is the mean of four similar determinations.

The amplitudes are those for the observed curves averaged as indicated above. The errors indicated for the different tides are the average differences from the means of the thirteen months, except in the case of the N–S diurnal tide, where it is simply an estimate. In combining the different tides to get a mean in each direction, the ratios and phase differences were weighted in proportion to the amplitudes of the tides except that the N–S diurnal tide was omitted. The final means of R and $\Delta\phi$ are the mean values obtained by weighting the five determinations in this way.

TABLE VII

	N–S			E–W		
	Amp.	R	$\Delta\phi$	Amp.	R	$\Delta\phi$
Lunar semi-diurnal...	6.188	0.6765 ± .019	1°15′ ± 1°2′	9.376	0.6915 ± .007	5°54′ ± 56′
Solar..............	3.150	0.716 ± .041	5°53′ ± 3°12′	4.429	.681 ± .018	5°30′ ± 1°28′
Lunar diurnal.......	.408	0.584 ± .200	(7°46′ ± 15°)	3.799	.698 ± .023	38′ ± 3°28′
Weighted means...	....	0.6895	2°41′	....	0.6903	4°34′
Final values	**R = 0.690 ± .004**			**$\Delta\phi$ = 4°**		

This seems to be the most logical procedure and is perhaps justified by the fact that the average difference from the mean for the different tides in each direction is roughly inversely proportional to the amplitude. The probable error, computed in the usual way, is given with the final value of R.

The final result indicates that the rigidity of the earth in the N–S and E–W directions is the same[1] and the ratio R is 0.690 with a probable error of ±0.004. That the viscous yielding of the earth is small is indicated by the small difference in phase between the observed and computed tides. It will be noted that the two solar tides appear to agree excellently in phase displacement with the E–W semi-diurnal tide and that for the N–S semi-diurnal, and probably also the E–W diurnal lunar tides the phase displacement is definitely smaller.

However, for lack of a better method of finding the means of the N–S and E–W phase displacement, each was averaged as the ratios were, that is, simply by weighting them in proportion to the amplitudes of the tides. This gives a displacement in phase of the water tides in the N–S direction of $+2°41'$ and in the E–W direction $+4°34'$. Although it seems certain that the difference in phase is slightly larger in the E–W than in the N–S direction, a mean displacement of $+4.°0$ is probably correct to within 1°. If we take $R=0.690$, the tides in the actual earth are 0.310 of what they would be if the earth were fluid, and the value of $\Delta\phi$ equal to $4.°0$, for the displacement of the water tides means that the earth tides lag behind the impressed forces by this same amount.

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[1] The preliminary experiment, through an error in computation, indicated a difference in the rigidities in the two directions. The ratio should have been 0.710 for both the N–S and E–W. See *Science*, October 3, 1919, p. 327.

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RYERSON LABORATORY
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